MILLIMETER WAVE CHARACTERISTICS OF FRESNEL ZONE PLATES

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ABSTRACT

Characteristics are given for Fresnel zone plates used as quasi-optical focusing or frequency-filtering elements at frequencies from 35 to 210 GHz. An analytical description of the far-field pattern has been developed. Design data and measured results are given for a new type of flat lens.

INTRODUCTION

The Fresnel zone plate is a planar device which has lens-like properties and can be used for focusing and imaging electromagnetic waves.[1,2,3] Zone plates accomplish these functions through diffraction and interference, rather than refraction. Since zone plates are relatively flat, they are simpler to construct than lenses and have the advantages of reduced thickness, weight, and absorption loss. There are two basic types of zone plates: one where alternate concentric annular zones are made opaque or reflecting and the second where phase correction is introduced in successive zones. Only the second kind will be considered here. This second case can be implemented using a single dielectric or with two or more different dielectrics.

For a single-dielectric zone plate, annular groves of varying depths produce the phase correction. In the multiple dielectric zone plate, different dielectrics of a constant thickness are used to provide the phase correction. This structure is called the planar lens. Figure 1 compares lenses and zone plates. In this paper the far field pattern for zone plates is derived and compared with results measured at millimeter wavelengths. For simplicity, zone plates considered in this paper will use two different phases to produce the desired phase correction. However, these derivations are easily extended to three or more phase zone plates. Second order effects, such as shadowing, will also be discussed. In addition design relations for zone plates will be given.

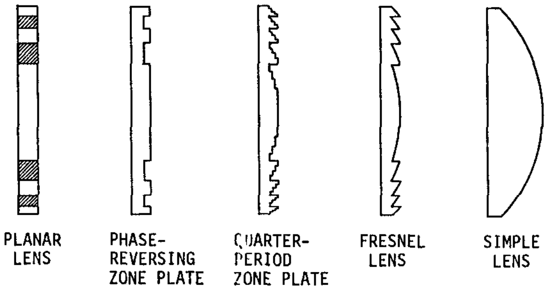

FIG. 1.-RELATION BETWEEN LENSES AND ZONE PLATES

Derivation of the Far-Field Pattern

The single-dielectric phase-correcting zone plate consists of a set of planar concentric annular rings cut into a flat piece of low-loss dielectric material, such as polystyrene. The successive radii of these zones are chosen so that the distance from a selected ("focal") point on the central axis increases by one-half wavelength in going from the inner to the outer radius of any ring, as illustrated in Figure 2. If a plane wave is normally incident on the zone plate, the portions of the radiation which pass through various parts of the transparent zones all reach the selected focal point with phases which differ by less than one half-period. Thus, the zone plate acts like a lens, producing a focusing action on the radiation it transmits.

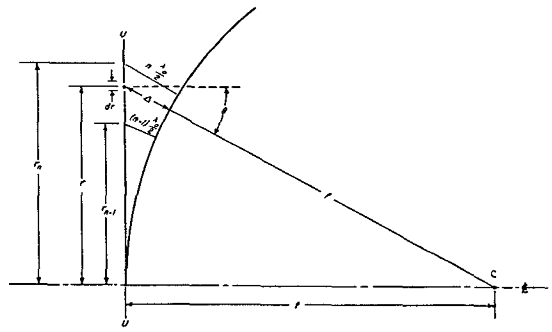

Fig. 2.-GEOMETRY OF THE n^{th} ZONE

The phase-corrected plate consists of a series of phased annuli together with one central aperture. To arrive at the far field pattern for the zone plate, the far field patterns for the

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individual annuli were superimposed. The first step was to derive a field expression for each annulus. Then the resulting annulus expressions were summed to obtain the far field pattern for the lens. The basic geometry used in the derivation is shown in Figure 3.

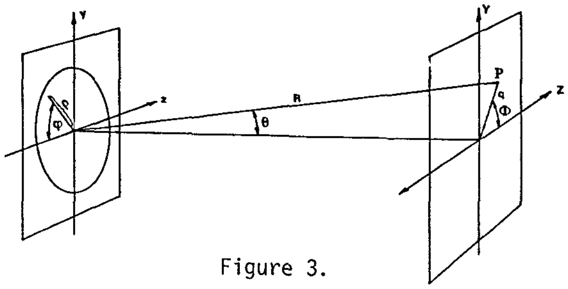


Figure 3.

Using the Kirchoff diffraction formula and including non-paraxial rays, the expression for the electric field produced by a circular aperture is given by:[4]

$$E(Y,Z) = (1 + \cos\theta) \iint_{aperture} \frac{e^{i[wt-kR]}}{R} e^{-i\delta n} F_{(y,z)} e^{\frac{ik}{R}(Yy + Z z)} dS$$

where $e^{-i\delta n}$ is the phase factor for the aperture and $F(x,y)$ is aperture illumination function.

Converting to cylindrical coordinates and letting F be a function of ρ only:[5]

$$E(\theta) = (1 + \cos\theta)\frac{e^{i(wt-kR)}}{R} e^{-i\delta n} \int_{r_{n-1}}^{r_n} F(\rho)\rho \int_0^{2\pi} e^{ik\rho \sin\theta \cos\phi} d\rho \, d\phi$$

where r_{n-1} is the inner radius of the aperture and r_n is the outer radius.

Note: $r_n > r_{n-1} \geq 0$

After performing the ϕ integration and breaking up the limit on ρ, $E(\Theta)$ becomes

$$E(\theta) = 2\pi (1 + \cos\theta)\frac{e^{i(wt-kR)}}{R} e^{-i\delta n} \left[\int_0^{r_n} F(\rho) J_0(k\rho \sin\theta)\rho d\rho + \int_0^{r_{n-1}} F(\rho) J_0(k\rho \sin\theta)\rho d\rho \right]$$

so the far field pattern problem can be reduced to solving the integral:

$$\int_0^a F(\rho)\ J_0(u\rho)\rho d\rho.$$

Of particular interest in the analysis of Fresnel zone plates is the case of spherical wave illumination. This case has been studied previously by Lommell[6] and Boivin[7] among others. Basically, Fresnel zone plates are designed to convert a spherical wavefront originating at the zone plate focus to an approximately planar wavefront or conversely to convert a planar wavefront to an approximately spherical wavefront converging to the focus.[3] By comparing the derived zone plate pattern for spherical wave illumination to the pattern obtained for a plane wave passing through an open aperture with the same dimension as the zone plate, the efficiency of the zone plate wave conversion can be analyzed and special features of zone plate patterns such as spikes in the antenna pattern can be explained.

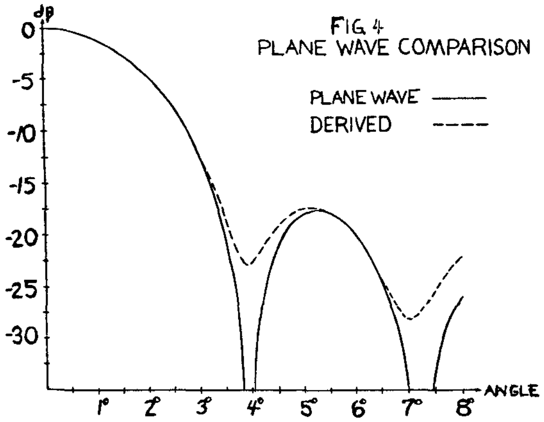
Using an illumination of $F(\rho) = e^{jk\sqrt{\rho^2+f^2}}$, $I(\theta)$ can be solved by first changing the variable ρ to $f\sqrt{r^2-1}$. Then the J_0 factor is written as a series. The resulting expression is integrated term by term.

$$I(\theta) = f^2 \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} \left(\frac{uf}{2}\right)^{2n} \int_1^{\sqrt{1+a/f^2}} e^{jKfr} r(r^2-1)^n dr$$

Using Euler's formula and the integral for a sine or a cosine multiplied by a polynomial,[8] this integration can be performed. Let $P^{(i)}_{2n+1}(r)$ be the i^{th} derivative of the 2n+1 degree polynomial $r(r^2-1)^n$.

$$I(\theta) = f^2 \left\{ \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} \left(\frac{uf}{2}\right)^{2n} \left[\frac{\sin Kfr}{Kf} \sum_{i=0}^{n} (-1)^i \frac{P^{(2i)}_{2n+1}(r)}{(kf)^{2i}} + \frac{\cos Kfr}{Kf} \sum_{i=1}^{n} (-1)^{i-1} \frac{P^{(2i)}_{2n+1}(r)}{(Kf)^{2i-1}} \right] \right\}$$
$$+ jf^2 \left\{ \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} \left(\frac{uf}{2}\right)^{2n} \left[\frac{-\cos Kfr}{Kf} \sum_{i=0}^{n} (-1)^i \frac{P^{(2i)}_{2n+1}(r)}{(Kf)^{2i}} + \frac{\sin Kfr}{Kf} \sum_{i=1}^{n} (-1)^{i-1} \frac{P^{2i}_{2n+1}(r)}{(Kf)^{2i-1}} \right] \right\}$$

evaluated at the limits of $\sqrt{1+(\frac{a}{f})^2}$ to 1. If uf > Kf, the above expression could be simplified by neglecting the higher order derivative terms. As a first order approximation, only the i=o derivative terms will be used. This approximation should hold if the $(\frac{uf}{2})^{2n} \frac{1}{(n!)^2}$ factor converges much faster than the derivative terms increase. For small angles this would probably be the case.



This form of analysis was applied as a test for the case of a plane wave (uniform phase and amplitude) across a circular aperture of the same size as the zone plate. Figure 4 compares the plane wave pattern with the pattern resulting from the derived expression. The result in Fig. 4 confirms the expected pattern, thus verifying the analysis. An observed characteristic of measured patterns is the presence of low-level (-30 dB) spikes in the 0-45^{0} range of the pattern. Analytically these spikes could possibly come from the additional derivative terms which must be included in the expression. These additional terms have an effect similar to adding higher order Bessel functions to the series at larger angles.

The far-field or diffraction patterns for Fresnel zone plates and their component annuli have been calculated previously by other authors.[4,8] Although both of these articles mentioned phase correcting zone plates, zone plates consisting of alternating opaque and transparent zones were mostly discussed. In addition these articles considered zone plates with focal lengths large enough so that $(\rho^2+f^2)^{1/2}$ can be approximated by $f+1/2\,\frac{\rho^2}{f}$ for phase terms and f for amplitude. Here ρ is the radial coordinate of the position on the zone plate. The pattern derivation in the present paper only requires f>o. For many applications a short focal length is desired.

Measured data for a zone plate from an earlier unpublished work will also be given. Angular deviation in the image, change in image distance behind the zone plate center, and the maximum signal level as a function of the angle θ were measured at 140 GHz using a 20 cm diameter, 12-zone half-period plate with a focal length equal to the diameter. The dimensions of this zone plate and comparable spherical and hyperboloidal lenses is illustrated in Figure 5.

With the single dielectric lens, there is the possibility of edge scattering and shadowing from the annular grooves. The effect of these phenomena on the zone plate far-field pattern will be estimated. These effects may be reduced by using the planar lens to introduce the desired phase correction. This lens has the geometrical advantage that the front and back surfaces are flat. Since the edges arising from annular grooves are absent, there should be less shadowing and scattering. In addition, the flat lens surface results in no accumulation of material on the lens and provides an aerodynamically smooth surface for easier use in airstreams.

Design formulas for both types of zone plate lenses will be given. The zone radius formula, $r_n = \sqrt{nf\lambda + (\frac{n\lambda}{2})^2}$, holds for both types of lenses. Groove depth as function of dielectric constant, ε, for the single dielectric lens is given by $d = \frac{\lambda_0}{2(\sqrt{\varepsilon}-1)}$.

Figure 6 provides a graph for selecting lens thickness and dielectrics for the multiple dielectric lens.

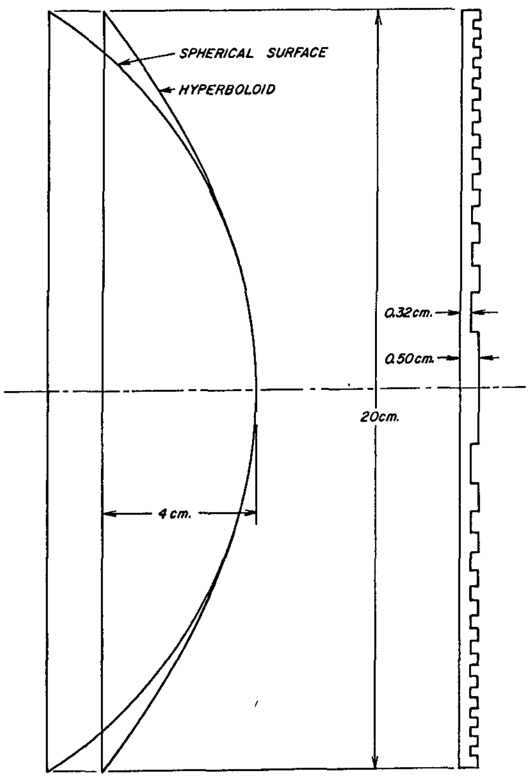


Fig. 5.-f/1 ZONE PLATE AND LENSES FOR 140 GHz. THE MATERIAL IS POLYSTYRENE.

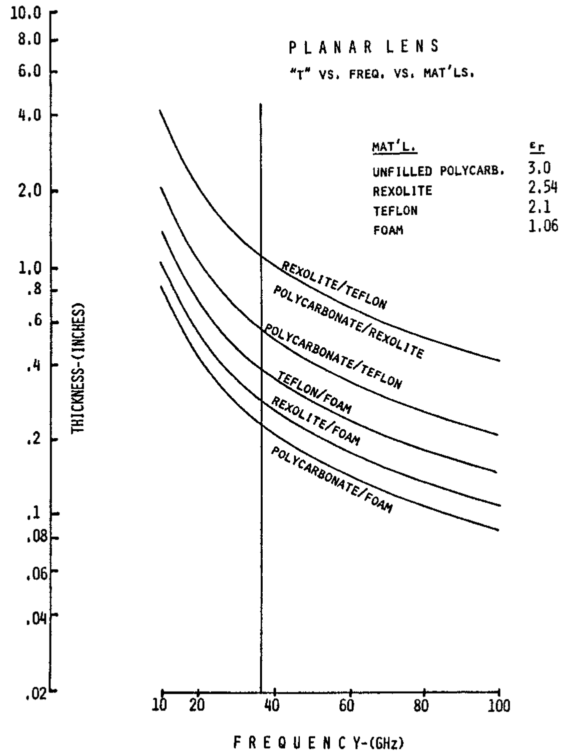


Figure 6.

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